The Fourth Triangle Area Graduate Mathematics Conference (TAGMaC)

Duke University, Department of Mathematics

April 23, 2017

This conference is sponsored by the Duke University AMS and SIAM graduate student chapters, and Information Initiative at Duke (iiD).

TAGMaC @Duke Schedule

8:30 - 10 (*Physics 101*) **Breakfast + Welcome**

10 - 12 **Session I**

Time \ Topic	Differential Equations I *Physics 205*	Computational Math *Physics 299*	Differential Geometry *Physics 227*	Combinatorics/Rep Theory *Physics 259*
10:00 - 10:20	Amanda Bernstein	Hangjie Ji	Gavin Ball	McKay Sullivan
10:20 - 10:30	break			
10:30 - 10:50	Erin Beckman	Sarah Ritchey	Yuhao Hu	Erika Ordog
10:50 - 11:10	break			
11:10 - 11:30	Brendan Williamson	Nick Battista	Henri Roesch	Seth Baldwin
11:30 - 11:40	break			
11:40 - 12:00		Ben Vadala-Roth		Michael Strayer

12-1:15 (*Physics 101*) **Lunch**

1:15 - 2:15 (*Physics 130*)
Plenary Talk: *Knot Invariants and Dualities.* David Rose, UNC

2:30 - 4:30 **Session II**

Time \ Topic	Differential Equations II *Physics 205*	Data/Statistics *Physics 299*	Algebraic Geometry *Physics 227*	Categorification *Physics 259*
2:30 - 2:50	Jacob Perry	Robert Ravier	Ma Luo	Dmitry Vagner
2:50 - 3:00	break			
3:00 - 3:20	Robert Booth	Ashleigh Thomas	Michael Ruddy	Victor Summers
3:20 - 3:40	break			
3:40 - 4:00	Katrina Morgan	Kevin Stubbs	Zachary Hough	Dan Scofield
4:00 - 4:10	break			
4:10 - 4:30		Daniel Irving Bernstein		Alex Chandler

4:30 - 6 (*Physics 101*) **Tea**

Plenary Talk: Knot Invariants and Dualities

David Rose, UNC - Chapel Hill

1:15 - 2:15 *Physics 130*

A knot is an embedding of the circle into three dimensional space, or in non-mathematician's terms, a knotted piece of string. These seemingly simple objects turn out to have deep connections to low-dimensional topology, algebra, and geometry. We'll discuss these basics, before turning our attention to quantum invariants, certain knot invariants introduced in the 80's and 90's using techniques in representation theory. These invariants admit an elementary description in terms of a certain diagrammatic calculus, and we'll explore a surprising (anti-)symmetry in this description. We'll then show how a certain algebraic duality explains this phenomenon and suggests new invariants and structure.

Differential Equations I: *Physics 205*

Real-Time Control for the Stabilization of a Double Inverted Pendulum

Amanda Bernstein, NCSU

10:00 - 10:20

We present the real-time implementation of two control strategies to balance a double inverted pendulum (DIP) on a cart. The DIP is considered to be stabilized when the two pendulums are aligned in a vertical position. The mathematical model for the dynamics of the DIP is derived using the Lagrange's energy method which is computed from the calculation of the total potential and kinetic energies of the system. This results in a highly nonlinear system of three second order ordinary differential equations. For our first control, we linearize the system around its zero equilibrium state and implement a linear quadratic regulator (LQR)-based controller to stabilize the system in real-time. Then we implement a controller based on the power series expansion of the Hamilton Jacobi Bellman (HJB) equation. Both simulation and real-time experimental results are presented.

Particle Systems and their PDE Limits

Erin Beckman, Duke

10:30 - 10:50

One of the things taught in an intro PDE course is that the solution to the heat equation can model the diffusion of perfume molecules in a room or of dye molecules in a glass of water. A key factor that allows us to draw this parallel between the motion of individual molecules and the solution of a PDE is the fact that these systems involve many particles. This behavior as the number of particles goes to infinity is called the hydrodynamic limit, and many particle systems have one. I will talk about the hydrodynamic limit and give examples of the limits for more complicated systems, which can be represented as solutions to various PDEs.

Controlling SDEs with partial dissipation

Brendan Williamson, Duke

11:10 - 11:30

If we consider a system of ODEs $dX_i = f_i(X(t))dt$ that conserves radius, it is easy to show that by adding Brownian noise dW_t and dissipation $-\gamma X_i(t)dt$ to each coordinate that the resulting SDE has a stationary measure. If however we add dissipation, and possibly noise, to a proper subset of the coordinates we have no such guarantees. We will show that for $f_i(x) = x^T C_i x$ with mild assumptions on the matrices C_i that one can prove the existence of a stationary measure. This problem is motivated by Stochastic PDEs with similar partial dissipation, in particular the inviscid Burgers and Navier Stokes Equations on S^1 and $S^1 \times S^1$ respectively.

Computational Mathematics: *Physics 299*

A general tear film model with locally elevated evaporation rates

Hangjie Ji, Duke

10:00 - 10:20

Motivated by a model proposed by Peng et al. [Advances in Coll. and Interf. Sci. 206 (2014)] for break-up of tear films on human eyes, we study the dynamics of a generalized thin film model. The governing equations form a fourth-order coupled system of nonlinear parabolic PDEs for the film thickness and salt concentration subject to non-conservative effects representing evaporation. We analytically prove the global existence of solutions to this model with mobility exponents in several different ranges and present numerical simulations that are in agreement with the analytic results. We also numerically capture other interesting dynamics of the model, including finite-time rupture-shock phenomenon due to the instabilities caused by locally elevated evaporation rates, convergence to equilibrium and infinite-time thinning.

Modeling blood flow with immersed interface method

Sarah Ritchey, Duke

10:30 - 10:50

Blood flow can be modeled as a fluid-structure interaction problem. Often, the blood vessel is represented by a closed tube immersed within a larger fluid domain. Flow is driven by a source and sink, placed at opposing ends of the tube. Deviation of the immersed interface from its equilibrium position generates a boundary force that is singularly supported on the interface. Due to this singular force, the pressure and the velocity gradient may be discontinuous across the interface. The immersed interface method incorporates the jumps in pressure and velocity into the finite difference approximations of the spatial derivatives in the discretized Stoke's equation. This closed-tube model is not ideal, since around the source and sink, the fluid does not behave like fluid flowing in a vessel. I will discuss the extension of the immersed interface method to model an immersed interface shaped like an open tube instead of a simple closed curve.

Fluid-structure interaction for the people!

Nick Battista, UNC

11:10 - 11:30

The development of fluid-structure interaction (FSI) software involves trade-offs between ease of use, generality, performance, and cost. Typically there are large learning curves when using low-level software to model the interaction of an elastic structure immersed in a uniform density fluid. Many existing codes are not publicly available, and the commercial software that exists usually requires expensive licenses and may not be as robust or allow the necessary flexibility that in house codes can provide. We present an open source immersed boundary software package, IB2d, with full implementations in both MATLAB and Python, that is capable of running a vast range of biomechanics models and was designed to be accessible to scientists who have experience in high-level programming environments. IB2d contains multiple options for constructing material properties of the fiber structure, as well as the advection-diffusion of a chemical gradient, muscle mechanics models, and artificial forcing to drive boundaries with a preferred motion. This talk will highlight some of its features and contextualize its functionality with current FSI literature.

Benchmarking an immersed finite element method for nearly-incompressible solid mechanics

Ben Vadala-Roth, UNC

11:40 - 12:00

A fundamental challenge in solving for the incompressible deformation of a solid body when using the finite element method (FEM) is finding a way to satisfy the Ladyzhenskaya-Babuska-Brezzi condition (LBB) while using solution spaces that are adequately regular for the physical problem at hand. Near incompressibility is imposed via a volumetric term in the strain energy, which functioning like a penalty method, prevents large compressible motions. Despite the LBB condition not being explicitly present in the nearly incompressible case, volumetric locking arises instead but is very similar to an LBB instability. Here we explore the effect of the penalty-like parameter in a standard benchmark problem but for the case when the velocities are interpolated from a fluid in a fluid-structure interaction (FSI) simulation. We then will show the empirical effects of imposing exact incompressibility on the fluid and how that may reduce the importance of using the penalty-like parameter and in turn avoid volumetric locking altogether.

Differential Geometry: *Physics 227*

Natural equations for closed G2-structures

Gavin Ball, Duke

10:00 - 10:20

A G2-structure on a 7-manifold M is a reduction of the structure group of the frame bundle of M to the exceptional Lie group G2. Equivalently, a G2-structure on M can be thought of as a 'nondegenerate', 'positive' 3-form on M. Such a 3-form induces a Riemannian metric on M. If the 3-form is closed and coclosed, this metric will have holonomy group a subgroup of G2. If the 3-form is closed but not coclosed, one can still write down some interesting nonlinear PDE for the G2-structure. In this talk I will explain the preceding 5 sentences, and outline a way to approach these equations using the theory of exterior differential systems and the moving frame.

The Geometry of Backlund Transformations

Yuhao Hu, Duke

10:30 - 10:50

A Bcklund transformation, roughly speaking, is a system of PDEs which enables one to start with knowing a solution of a PDE system (A), and using ODE techniques alone, to find parametrized solutions of a PDE system (B). The meaning of "geometry" for Bcklund transformations is two-fold. One is that there are quite a few Bcklund transformations that connects PDE systems bearing geometric meanings such as those for surfaces with prescribed curvature(s) inside a certain 3-dimensional homogeneous space. The other is that Bcklund transformations themselves can be understood as manifolds endowed with geometric structures, so that it is natural their geometric invariants. In this talk, I'll give the definition of a Bcklund transformation, some examples, then an outline of the geometric invariant approach, and, finally, discuss some open problems in this direction.

Null Geometry and the Penrose conjecture

Henri Roesch, Duke

11:10 - 11:30

In the theory of general relativity, the Penrose conjecture claims a lower bound for the mass of a spacetime in terms of the area of an outermost horizon, if one exists. In physical terms, this conjecture is a geometric formulation of the statement that the total mass of a spacetime is at least the mass of any black holes that are present, assuming non-negative energy density. For the geometry of light-rays emanating off of a black hole horizon (called a nullcone), the Penrose conjecture can be reformulated to the so-called Null Penrose Conjecture (NPC). In this talk, we will explore key aspects of null geometry, leaning on example nullcones to describe recent results on the NPC.

Combinatorics/Representation Theory: *Physics 259*

Supersymmetric Bilinear forms and Oscillator Algebras

McKay Sullivan, NCSU

10:00 - 10:20

We introduce inhomogeneous supersymmetric bilinear forms on a complex superspace and show that they lead to oscillator-like superalgebras where the products of bosonic oscillators with fermionic oscillators are not necessarily zero. We give a classification for superspaces up to dimension 7, provide examples, and indicate a possible approach to generalize the classification to any finite dimension. This talk is based on joint work with Bojko Bakalov.

Canonical Minimal Free Resolutions of Monomial Ideals in Three Variables

Erika Ordog, Duke

10:30 - 10:50

A monomial ideal in three variables can be resolved by a planar map embedded in the staircase surface for that ideal, but the planar map is often not unique. In this talk, I will explain how we can use the Moore-Penrose pseudo-inverse to obtain a differential that, in a way, averages the minimal free resolutions to give one that is canonical. I will also explain the combinatorial object underlying the computation of this differential map, which allows us to write down the canonical minimal free resolution by looking at the staircase surface. Coauthors: John A. Eagon and Ezra Miller.

K-theoretic Schubert calculus of the affine Grassmanian associated to SL_2.

Seth Baldwin, UNC

11:10 - 11:30

Schubert calculus is a branch of algebraic geometry introduced to solve various counting problems in projective geometry. The objects of study are certain locally closed subsets of Grassmanians, called Schubert cells, defined by conditions of linear incidence. A modern approach to Schubert calculus involves studying the product structure in the associated singular cohomology ring to a Grassmanian, where a basis is given by the classes of the Schubert cells. In this talk, after developing the appropriate motivation, we will discuss a K-theoretic analog of Schubert calculus in the case of affine Grassmanians. In particular we will consider the example of the affine Grassmanian associated to SL_2, where we explicitly determine the multiplication in K-theory.

Combinatorially described Demazure modules for simply-laced Kac-Moody algebras

Michael Strayer, UNC

11:40 - 12:00

In 2001 N.J. Wildberger published a paper on a method to construct minuscule representations of certain finite-dimensional Lie algebras using combinatorial techniques related to the partially ordered set of weights of such a representation. In this talk we present an extension of this idea to simply-laced Kac-Moody algebras, and in particular describe necessary conditions on the colored poset structure to obtain a ”minuscule” representation of the Borel subalgebra acting on some lowest weight vector; that is, a Demazure module. This new notion of ”minuscule” in the Kac-Moody setting will be described, as will connections to the work of J.R. Stembridge in the late 1990s.

Differential Equations II: *Physics 205*

Localized energy estimates for wave equations with trapping, part 1

Jacob Perry, UNC

2:30 - 2:50

When studying wave equations on differentiable manifolds, it is known that geodesic trapping necessitates a loss in standard local energy estimates. For non-degenerate hyperbolic trapping the loss is logarithmic, while for elliptic trapping everything is lost except a logarithm. In our work, we consider an asymptotically flat manifold with degenerate hyperbolic trapping. This talk will develop a local energy estimate that holds everywhere except on the trapped set, via separate analysis of high and low frequency behavior. This estimate allows for local analysis near the trapped set, where algebraic loss can be attained. This talk is part of a joint work with Robert Booth, Hans Christianson, and Jason Metcalfe.

Localized energy for wave equations with degenerate trapping

Robert Booth, UNC

3:00 - 3:20

When studying wave equations on differentiable manifolds, it is known that geodesic trapping necessitates a loss in standard local energy estimates. For non-degenerate hyperbolic trapping, the loss is logarithmic. For elliptic trapping, everything is lost except a logarithm. In our work, we consider a manifold with degenerate hyperbolic trapping, attaining an algebraic loss. This talk will utilize a first weaker estimate that is interesting in its own right. We will focus on the use of a WKB inspired analysis to attain the sharp result and prove sharpness via a quasimode construction. This is joint work with Hans Christianson, Jason Metcalfe, and Jacob Perry.

Analytic Tools for Establishing Pointwise Decay Rates of Waves on Stationary Space-Time Geometries

Katrina Morgan, UNC

3:40 - 4:00

My research focuses on determining the pointwise decay rate of waves on a specific class of stationary space-time geometries. Important techniques for this work include resolvent estimates, commutator arguments, and Fourier analysis. At their core many of these ideass can be explained in terms of elementary concepts like the analysts favorite tool, integration by parts. The purpose of this talk is to provide an overview of some of the arguments used to obtain pointwise decay rates of waves in the context of more elementary ideas.

Data/Statistics: ***Physics 299***

Eyes on the Prize: Synchronization through Forward Propagation

Robert Ravier, Duke

2:30 - 2:50

Synchronization problems seek to align a collection of objects in a coherent manner, where the words align and object have rather broad meanings. A difficult variant of this problem is shape matching, where we seek to find accurate correspondences between different points on shapes in a consistent (e.g. commutative) manner. As a step towards a solution to both shape matching and more general synchronization problems, it was shown in the 2013 thesis of Jesus Puente that, provided that the maps are generated as minimizers of a distance functional, the problem can be solved with reasonable accuracy via the use of a minimum spanning tree (MST). The MST based method is highly prone to accuracy and stability issues, as the higher dimensionality of the space of shapes can result in wildly different spanning trees when more data is added to the system. In this talk, we outline a generalization of the MST approach by constructing a significantly more stable probabilistic algorithm that only considers paths meeting natural geometric criteria. We will then show by example that this method can improve correspondences between similar shapes, accurately register existing corresponding points on dissimilar shapes, and even tell us when a point on one shape does not have a corresponding point on another.

Algebraic statistics for fly wing evolution

Ashleigh Thomas, Duke

3:00 - 3:20

We use two-parameter persistent homology to summarize the morphology of fly wing veins for an evolving population of flies. We hypothesize that topological novelty in vein morphology arises when directional selection pushes continuous variation in a developmental program beyond a certain threshold. In this case, the directional selection is for certain continuous wing deformations and the resulting topological novelty is added/missing pieces/intersections of veins. For this data set, persistent homology outputs infinitely generated modules over a non-Noetherian ring. We study finiteness conditions and computable presentations of these modules. This work is joint with Florida State University biologist David Houle and Duke mathematicians Ezra Miller, Justin Curry, and Surabhi Beriwal.

Non-linear Representations for TEM Crystal Image Analysis

Kevin Stubbs, Duke

3:40 - 4:00

Defects play a fundamental role in the physical properties of polycrystalline materials. Even small deviations from a perfect crystalline structure can drastically change the macroscopic properties of a material. With the advent of transmission electron microscopy (TEM), scientists have become able to take atomic level pictures of crystals and study these defects directly. Since defects in crystals occur when symmetry is broken, it is an interesting mathematical question to create a robust algorithm which can use TEM images to calculate physical information about the defects (i.e. location, orientation, etc.). In this talk, I will look at two non-linear methods for locating defects in crystals; one which to extracts all information about a crystal and one which uses more statistical techniques. Non-linearity seems to play a key role in ensuring both techniques are stable under the types of deformations which occur in real world images.

Combinatorial algebraic geometry in low-rank matrix completion

Daniel Irving Bernstein, NCSU

4:10 - 4:30

Given a matrix where only some of the entries are known, is it possible to recover the missing entries if we assume the matrix has some low rank r? This problem has many applications including the "Netflix problem" so it has been studied quite a bit by engineers. I will discuss how ideas from combinatorial algebraic geometry can be useful here.

Algebraic Geometry: *Physics 227*

Periods and Galois Theory

Ma Luo, Duke

2:30 - 2:50

Periods are numbers that can be written as integrals of algebraic quantities. They form a countable ring, which includes all algebraic numbers, and some transcendental numbers such as π, multiple zeta values and elliptic integrals. We will discuss how one can develop a Galois theory of all periods, which was envisioned by Grothendieck long time ago via his period conjecture. Recently, there has been plenty of development in this direction, most notably Brown's work, which provides us a Galois theory of (motivic) multiple zeta values.

Signature Map on Algebraic Curves

Michael Ruddy, NCSU

3:00 - 3:20

Let G be a Lie group acting smoothly on the plane. Then two smooth curves C and C' are G-equivalent if there exists some g in G such that g*C=C'. Can we answer the question, when are two curves G-equivalent? What can we additionally say if we restrict our attention to algebraic curves? In this talk we will first introduce the signature map and explain how it can help identify when two smooth curves are G-equivalent. Then we will investigate the signature map on algebraic curves and see what additional information we can glean. This is joint work with Drs. Irina Kogan and Cynthia Vinzant.

Degree-optimal moving frames for rational curves

Zachary Hough, NCSU

3:40 - 4:00

I will present an algorithm that, for a given vector of n relatively prime polynomials in one variable over an arbitrary field, outputs an invertible matrix with polynomial entries such that it forms a degree-optimal moving frame for the rational curve defined by the input vector. From an algebraic point of view, the first column of the matrix consists of a minimal-degree Bezout vector (a minimal-degree solution to the univariate effective Nullstellensatz problem) of the input vector, and the last n-1 columns comprise an optimal-degree basis, called a mu-basis, of the syzygy module of the input vector. The algorithm and underlying theory are based on elementary linear algebra. Degree bounds for the degree-optimal moving frame and minimal-degree Bezout vector will be discussed as well.

Categorification: *Physics 259*

A Smooth TQFT approach to $\mathfrak{sl}_n$ Homology

Dmitry Vagner, Duke

2:30 - 2:50

Given a link diagram L, one can apply a Skein relation to each crossing to yield a cube of resolutions. These skein relations come from the braiding in the category of $U_q(\mathfrak{sl}_n)$ representations. When $n = 2$, we have the Khovanov cube of resolutions with edge maps defined by (co)pants cobordisms. We may then apply a smooth TQFT, flatten the complex, and take homology to extract Khovanov homology. The case is more complicated when $n > 2$ because, due to the higher complexity of morphisms among fundamental representations, the resolution diagrams will contain trivalent vertices. The first definition, due to Khovanov and Rozansky, defined the invariants in the category of matrix factorizations. A more diagrammatic and combinatorial approach, taken by Queffelef-Rose, considers foam cobordisms—those with seams—between trivalent graphs and exploits skew-Howe duality to define a foam-like TQFT on them so as to extract homology. In this work, joint with Michael Abel, we apply a virtual filtration—another Skein-like relation, justified by the behavior of matrix factorizations—to resolve all trivalent graphs to their own cubes of smooth resolutions. With a little work, this yields a triple complex from which we extract a spectral sequence whose second page is equivalent to the Khovanov-Rozansky $\mathfrak{sl}_n$ homology of the link.

Magnitude Homology

Victor Summers, NCSU

3:00 - 3:20

Many mathematical constructions come equipped with a canonical measure of size; the cardinality of a set, Euler characteristic of a topological space, dimension of a vector space, to name just three. Tom Leinster added magnitude of a metric space to this list of cardinality-like invariants. Magnitude extends more generally to enriched categories of which metric spaces are a particular example. While the definition of magnitude is highly abstract, it turns out to be mysteriously rich in geometric information. Graphs can be regarded as metric spaces; as such they have magnitude. Richard Hepworth and Simon Willerton went on to categorify the magnitude of graphs, realizing the invariant as the Euler characteristic of a bigraded homology theory. I will describe magnitude of graphs along with its categorification as well as some recent results regarding this homology theory.

Torsion in Khovanov homology via chromatic graph cohomology

Dan Scofield, NCSU

3:40 - 4:00

The categorification of the chromatic polynomial by Helme-Guizon and Rong is isomorphic to Khovanov link homology over a range of homological gradings. Motivated by Hochschild homology, we compute torsion in chromatic homology for certain classes of graphs. As a consequence, we offer insight into Z_2 torsion of certain classes of knots and links.

A Categorification of the Vandermonde Determinant

Alex Chandler, NCSU

4:10 - 4:30

Given a knot diagram D with n crossings and a list of natural numbers $x_1, ..., x_n$, we will construct a commutative diagram of smoothings and cobordisms. After applying a TQFT to this diagram and flattening, we get a chain complex. Choosing D to be the alternating braid diagram of the $(2, n)$ torus knot, we find that the Euler characteristic of the complex is equal to the Vandermonde determinant in the variables x_i.
